# Computer programming in the professional development of future mathematics teachers


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## Introduction and Aim

With the growing integration of programming in mathematics classrooms, we see a crucial need to understand how university students and future mathematics teachers may come to appropriate programming as an instrument (Trouche, 2004) for themselves –to carry out programming-based mathematical investigations–, as well as to design teaching and learning resources for others.

- A **5-year research study**, funded by the Canadian Social Sciences and Humanities Research Council (SSHRC)
- Aim of the study: Examine how postsecondary mathematics students learn to use programming as a computational thinking instrument for mathematics.
- A naturalistic case study taking place in the context of a sequence of 3 programming-based (MICA) mathematics courses implemented in the mathematics department at Brock University (Canada), since 2001
- University students majoring in mathematics and **future mathematics teachers** learn to design, program, and use interactive computer environments to investigate mathematics conjectures, concepts, theorems, or real-world applications.
- Poster's focus: **How future teachers use programming for their learning and in teaching resources they develop**.

## Context: Brock University's *Mathematics Integrated with Computers and Applications* (MICA) program (cf Buteau & Muller, 2010)

- Three one-semester courses, over three years:
  MICA I, II and III/III* –III for math & science majors / III* for preservice teachers.
- 14 programming-based mathematics investigation projects (in VB.Net, Python or, in MICA III*, also Scratch)
  - In each course: 3 or 4 predefined EOs*
  - + end-of-term final project (carried out individually or small teams):
    - In MICA I, II, III: student-selected topics for a final project EO or LO** .
    - In MICA III*: Design of a teaching resource of programming-based mathematics activities. In analyzed data: in accordance with Ontario's regional curriculum, using as model the UK's ScratchMaths (UCL, 2018) curriculum and pedagogy; should include: tasks using Python programming, worksheets in Jupyter Notebook, a short video and follow-up resources for teachers (and optional additional resources, e.g., in Scratch).

* *Exploratory Object* (EO) –microworld-type interactive computer-based model to explore a mathematical concept, conjecture, or real-world situation.

***Learning Object* (LO): a computer-based learning interactive object (game or activity) to guide a learner step-by-step towards understanding a school mathematical concept

- Analyses of MICA's approach indicate that it promotes experiences that may facilitate the appropriation of programming as an instrument for mathematical inquiry (Buteau et al., 2016).

## Theoretical framework

- The Instrumental Approach: an artefact becomes an instrument through the dual processes of instrumental genesis – instrumentation and instrumentalization. An instrument is (parts of) the artefact together with a scheme of use. (Trouche, 2004)
- The Documentational Approach to Didactics (Trouche et al., 2018).
  - An extension of the instrumental approach, related to teachers' professional development.
  - Focus on how mathematics teachers interact and use resources (including the digital ones), through the (re-)design or 'design-in-use' of resources for their own work and/or the collective work with other teachers.
  - Resources can be material (e.g., textbooks, digital resources, manipulatives, tasks), social and cognitive (e.g., frameworks/theoretical tools used in work with teachers).
  - When teachers interact with resources, they change and develop their professional knowledge; this is called the teacher's *documentation work.*

## Methodology

- Mixed methodology and iterative design approach
- Data collected include each participant's LO, their associated report, their final projects which are teaching lessons, and semi structured interviews with them.

## Background

- In our larger research, we have been using the instrumental approach to analyse how MICA participants develop programming from an artefact into an instrument (Gueudet & Trouche, 2009) that they can integrate into their practice as mathematicians (and teachers); and have analysed their instrumented schemes.
- In Mgombelo et al. (in press), we inferred some instrumental schemes that future teachers develop during the development process of an LO.

## The various geneses for future teachers through MICA

- During MICA, future teachers undergo instrumental and documentational geneses, to appropriate programming for themselves and for teaching. The initial artefact (or resource) of programming can become both a personal instrument for mathematical work (, as well as a didactic instrument in the professional teaching activity.
- It starts with students creating EOs (which begins in MICA I but continues up to MICA III/III*) and developing schemes that allow them to appropriate programming as an instrument for mathematical inquiry. Future teachers then develop further their instrumental geneses by creating LOs. Later, in designing the final MICA III* teaching resources they learn how to integrate programming with didactic intentions.
- In that work, the resource of programming needs to interact with other elements (other resources, that together with programming constitute a system of resources) –a process of documentational genesis. (These processes are schematised in Figure 1).

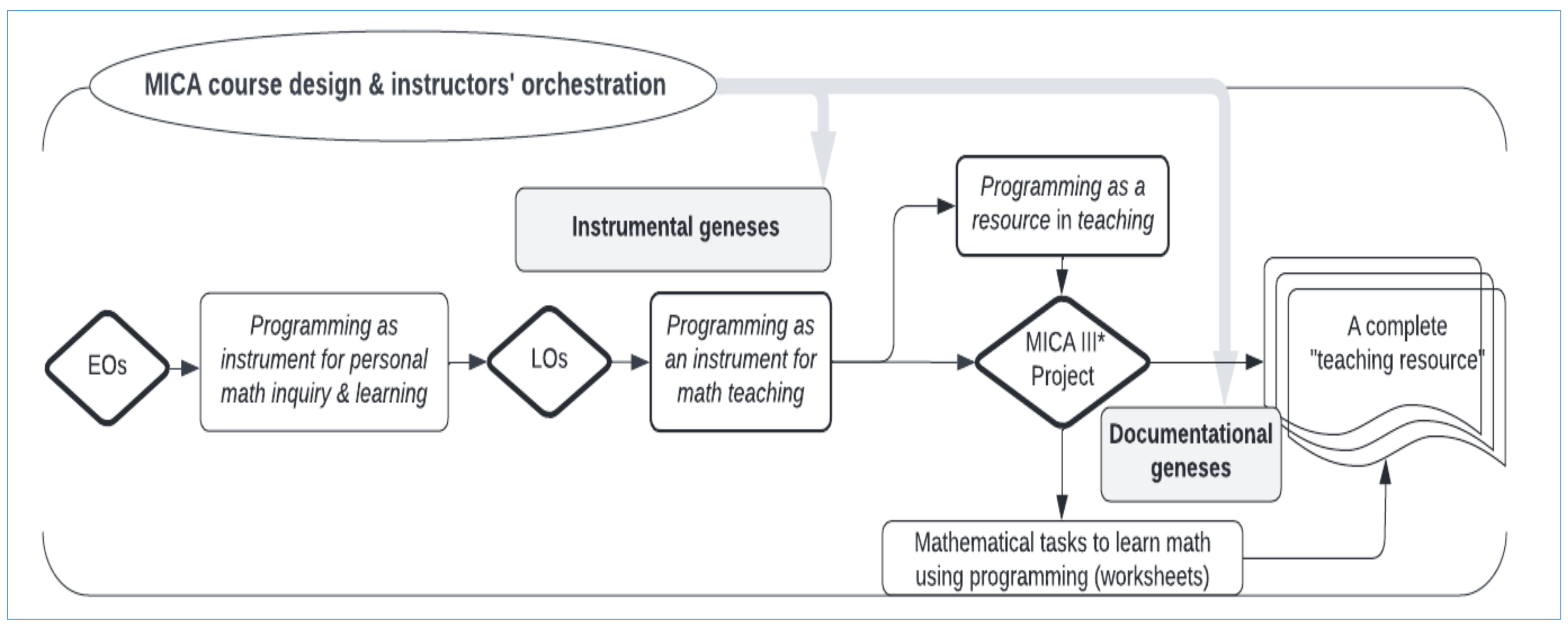
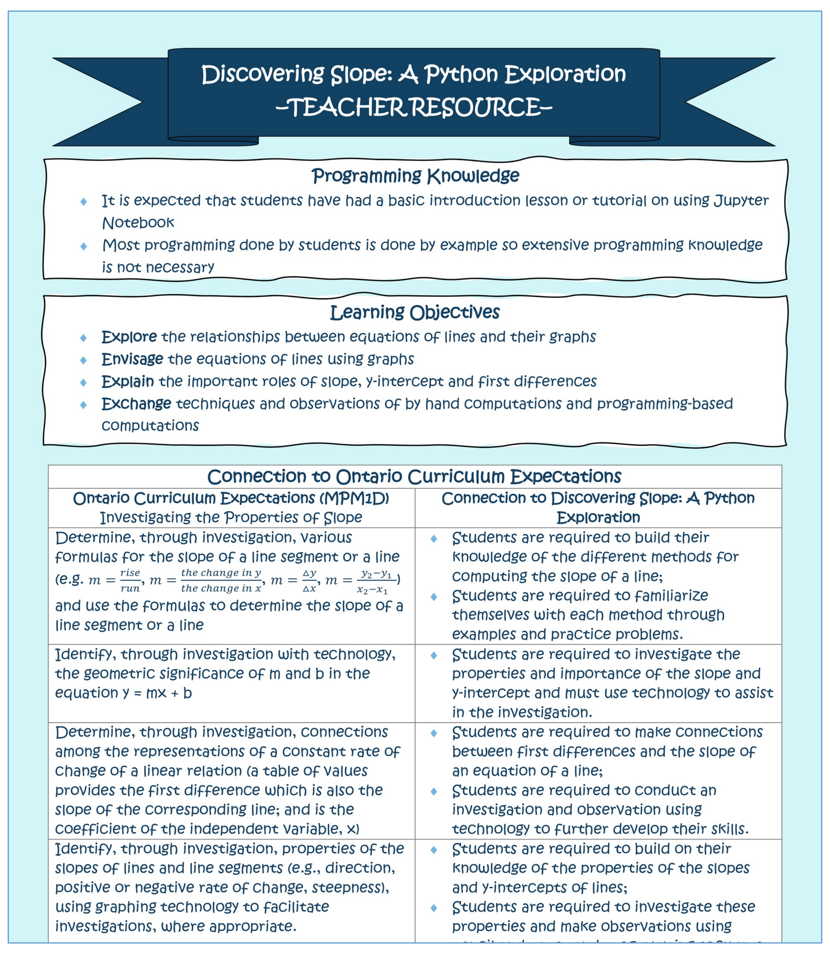



**Figure 1. Future teachers' instrumental and documentational geneses in the MICA programme**



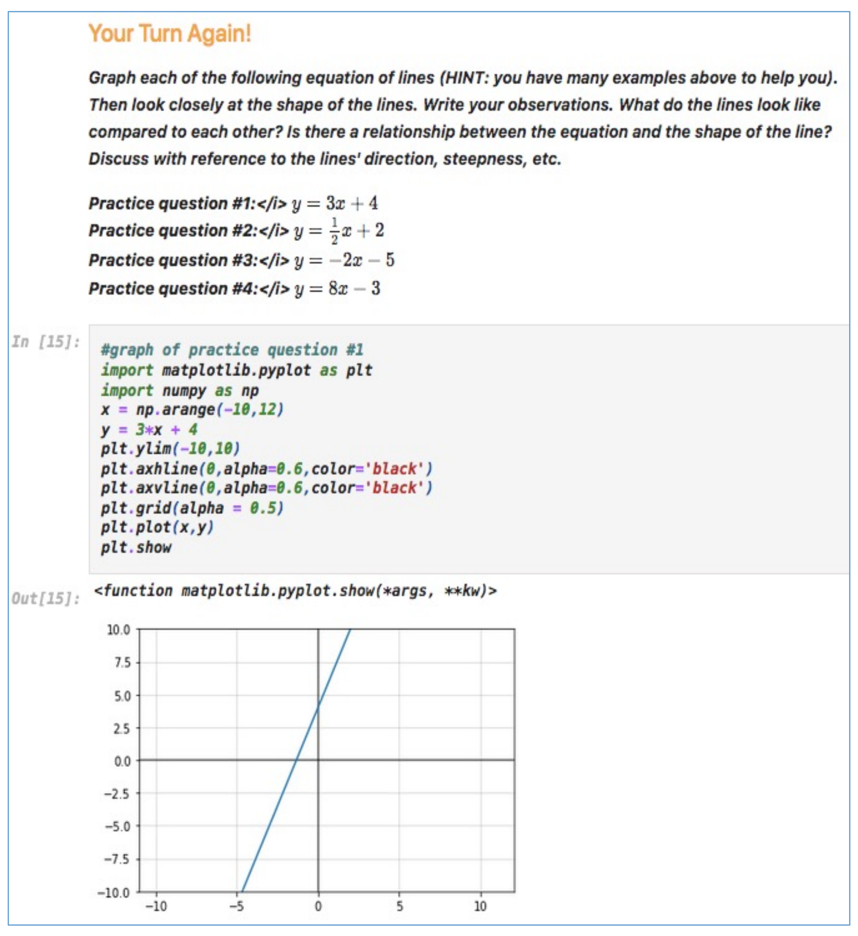
Discovering Slope: A Python Exploration
-TEACHER RESOURCE-

Programming Knowledge
- It is expected that students have had a basic introduction lesson or tutorial on using Jupyter Notebook
- Most programming done by students is done by example so extensive programming knowledge is not necessary

Learning Objectives
- **Explore** the relationships between equations of lines and their graphs
- **Envisage** the equations of lines using graphs
- **Explain** the important roles of slope, y-intercept and first differences
- **Exchange** techniques and observations of by hand computations and programming-based computations

Connection to Ontario Curriculum Expectations

| Ontario Curriculum Expectations (MPM1D) Investigating the Properties of Slope | Connection to Discovering Slope: A Python Exploration |
|---|---|
| Determine, through investigation, various formulas for the slope of a line segment or a line (e.g. $m = \frac{rise}{run}$, $m = \frac{\text{the change in } y}{\text{the change in } x}$, $m = \frac{\Delta y}{\Delta x}$, $m = \frac{y_2 - y_1}{x_2 - x_1}$) and use the formulas to determine the slope of a line segment or a line | • Students are required to build their knowledge of the different methods for computing the slope of a line; • Students are required to familiarize themselves with each method through examples and practice problems. |
| Identify, through investigation with technology, the geometric significance of m and b in the equation y = mx + b | • Students are required to investigate the properties and importance of the slope and y-intercept and must use technology to assist in the investigation. |
| Determine, through investigation, connections among the representations of a constant rate of change of a linear relation (a table of values provides the first difference which is also the slope of the corresponding line; and is the coefficient of the independent variable, x) | • Students are required to make connections between first differences and the slope of an equation of a line; • Students are required to conduct an investigation and observation using technology to further develop their skills. |
| Identify, through investigation, properties of the slopes of lines and line segments (e.g., direction, positive or negative rate of change, steepness), using graphing technology to facilitate investigations, where appropriate. | • Students are required to build on their knowledge of the properties of the slopes and y-intercepts of lines; • Students are required to investigate these properties and make observations using |



Your Turn Again!

*Graph each of the following equation of lines (HINT: you have many examples above to help you). Then look closely at the shape of the lines. Write your observations. What do the lines look like compared to each other? Is there a relationship between the equation and the shape of the line? Discuss with reference to the lines' direction, steepness, etc.*

**Practice question #1:</i>** $y = 3x + 4$
**Practice question #2:</i>** $y = \frac{1}{2}x + 2$
**Practice question #3:</i>** $y = -2x - 5$
**Practice question #4:</i>** $y = 8x - 3$

In [15]:
```
#graph of practice question #1
import matplotlib.pyplot as plt
import numpy as np
x = np.arange(-10,12)
y = 3*x + 4
plt.ylim(-10,10)
plt.axhline(0,alpha=0.6,color='black')
plt.axvline(0,alpha=0.6,color='black')
plt.grid(alpha = 0.5)
plt.plot(x,y)
plt.show
```

Out[15]:
```
<function matplotlib.pyplot.show(*args, **kw)>
```

**Figure 2. Sample MICA III* Final Project**
**a. "Teacher Resource" summary (p. 1) b. Python task and code from teachers' solution key**

## Documentational genesis in a MICA III* final project

- We identify elements of the documentational genesis of future teachers:, they appropriate programming as a resource when they create LOs and in designing the final teaching lessons.
- Creating LOs involves a certain degree of documentational genesis. But the final MICA III* project requires deeper interactions of diverse resources and knowledge (including that of programming for math). The "teacher resources" in the project are systems of resources with usage schemes, i.e., documents– that they develop for teachers.
- *Sample data*: Barbara and her partner developed a teacher resource for exploring slopes of functions with Python. That resource included the activity summary of the "Teacher Resource" (Figure 2a), accompanying self-contained Python worksheets for students with solution file for the teacher (Figure 2b), and additional resources that include a required video, as well as posters, and cards for additional activities. The activity summary contains clear learning objectives in terms of the math content, alignment to the curriculum, as well as "the five Es" of the ScratchMaths pedagogy, activity instructions, discussion points, things to note, etc. The teaching resource includes programming tasks for students, to be carried out by following Python code examples, so that extensive knowledge would not be needed, with the code being re-used and modified.
- Sample analysis of Barbara's case: In her documentational genesis, one of her schemes, with the goal (for her teaching resource) of designing math tasks that integrate coding, was based on a previous programming and math (p+m) scheme (Gueudet et al., 2022) developed through a personal instrumental genesis, that now extended to the new situation of designing teaching tasks, which require professional knowledge (e.g., curricular considerations) in interaction with her programming (for math) knowledge.

## Concluding notes

During the MICA activities, future teachers develop personal instrumental geneses of computer programming for mathematical investigations, as well as the dual processes of documentational genesis: Their instrumentalization in terms of how programming affordances influence their task design for how the programming-based teaching resources that they design, can scaffold other students' activity. And their instrumentation involving schemes within the design of the teaching resource; for instance, a scheme that guides their task design provides future teachers with the opportunity to interact with a wide and diverse system of resources for their teaching: computer programming, curricular resources, worksheets, specialised software, etc.

## References


Buteau, C. & Muller, E. (2010). Student development process of designing and implementing exploratory and learning objects. In V. Durand-Guerrier, S. Soury-Lavergne & F. Arzarello, (Eds.), *Proceedings of CERME 6*, 2009 (pp. 1111–1120). Lyon, France: INRP. http://www.inrp.fr/editions/cerme6

Buteau, C., Muller, E., Marshall, N., Sacristán, AI., Mgombelo, J. (2016). Undergraduate mathematics students appropriating programming as a tool for modelling, simulation, and visualization: A case study. *Digital Experiences in Mathematics Education*, 2(2), 142–166.

Gueudet, G., Buteau, C., Muller, E., Mgombelo, J., Sacristán, A. I. & Santacruz-Rodríguez, M. (2022). Development and evolution of instrumented schemes: A case study of learning programming for mathematical investigations. *Educational Studies in Mathematics*, 110(2), 353–377.

Mgombelo, J., Pinar, A., Buteau, C., Muller, E., Sacristán, A. & Santacruz, M. (In press). Pre-service teachers using programming to design learning objects: A case study. *Proceedings. CERME 12,* Bozen-Bolzano.

Trouche, L., Gueudet, G. & Pepin, B. (2018). Documentational Approach to Didactics. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education* (pp. 1–11). Springer.

Trouche, L. (2004). Managing the complexity of human/machine interactions in computerized learning environments: Guiding students' command process through instrumental orchestrations. *International Int J Comput Math 9*(3), 281–307. https://doi.org/10.1007/s10758-004-3468-5

UCL (2018). *UCL ScratchMaths Curriculum*. IOE, University College London. https://www.ucl.ac.uk/ioe/research/projects/ucl-scratchmaths/ucl-scratchmaths-curriculum



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