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Coding, Computational Modeling, and Equity in Mathematics Education

P.D. Day & Symposium
April 26, 27-29, 2023

FIELDS Brock University Western University · Canada Callysto SSHRC CRSH

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In mathematics classrooms across the globe, educators are increasingly integrating various forms of plugged-in and unplugged coding activities with the interest of promoting knowledge and skills related to computational thinking (CT). Plugged-in coding activities are taught via block-based and text-based computer programming software such as Scratch and Python, respectively. Unplugged coding activities typically involve screenless methods, including paper instructions, game-based learning, manipulatives, robotics etcetera. The various forms of practical applications in mathematics classrooms arise from increased research in CT, promulgated by societal changes. Certain societal changes are due to the rapid pace of technological developments, and these necessitate learner's computational participation in areas such as artificial intelligence, robotics, and coding. Notwithstanding, the concept of CT remains somewhat elusive, giving rise to ongoing concerns about equity in mathematics education and the extent to which K-12 teachers are prepared to implement coding-focused mathematics curricula, and artificial intelligence-focused mathematics curriculum.

The Professional Development (PD) day and Symposium on Coding, Computational Modeling, & Equity in Mathematics Education (CCMEME), hosted by Brock University on April 26, 27- 29, 2023, was an effort to address such emerging issues highlighted by research. The PD day and symposium involved workshops, working groups, plenary, panel discussions, and reactions. The organization of the event brought together two SSHRC-funded research teams hosted at Brock University (led by Dr. Chantal Buteau) and Western University (led by Dr. Immaculate Namukasa), who work on programming and computational participation across all levels of mathematics education (PreK-16).

The complimentary PD on April 26 was a precursor to the symposium, with pre-registration numbers over 100, and participation numbers over 60 in person participants including pre-service and in-service teachers, educators, and young and experienced researchers. The hands-on nature of the variety of plugged-in and unplugged coding workshops, facilitated by a range of educational partners and educators across Canada, offered practical suggestions of how teachers and mathematics educators of all levels may integrate coding and computational modeling activities into CT-focused mathematics curricula. Workshop topics included, but were not limited to, Artificial Intelligence and Scratch, Integrating Coding in the Curriculum, Incorporating Data Visualization Exercises, Introduction to AI for Educators, and Introduction to Thinking Classrooms, and Robotics.

The symposium that followed on April 27- 29, 2023 registered 63 educators, scholars, graduate students and other stakeholders of diverse backgrounds in-person, with other participants attending the online sessions. Attendees, presenters and panelists spanned three working groups, representing the United States, Mexico, Brazil, France, Norway and Australia and provinces from across Canada. Additionally, virtual presenters and panelists represented France, the United Kingdom, the Netherlands, and South Korea. Of the three working groups, two focused on coding and computational modeling in mathematics education (one group for elementary and early years, another group for secondary and post-secondary), and one group focused on equity, diversity and inclusivity in coding and computational modeling. Over the three days of the symposium, attendees spent several hours with their working groups sharing ideas, participating in activities and generating knowledge together.

The plenary sessions, panel discussion and the three working groups facilitated activities and discussions supporting the following themes of the symposium:

1. Meanings and interactions between “computational thinking” and “mathematical thinking.”
2. Integration of coding and computational modeling in math curricula and classrooms at different levels of education.
3. Theoretical and practical understandings related to improving equity in integration of CCM in mathematics education.

On the first evening of the symposium, Dr. Andrea diSessa, Professor Emeritus in Education at UC Berkeley, amalgamated years of experience, research and knowledge in his keynote presentation titled “Five Powerful Ideas about Technology and Education.” Dr. diSessa synthesized 40 years of work in computers and education, presenting a concise list of five essential ideas crucial for progress in education. Given the extended developmental and uncertain trajectory of computers - a once in several centuries innovation - he emphasized the necessity of these ideas.

Firstly, Dr. diSessa asserted that beyond teaching coding, we should embrace a new literacy based on computational media, intended to augment rather than abandon the traditional literacy based on text. He argued that developing classic literature and the social infrastructure for computation literacy takes time, but it is necessary to avoid stagnation in the past or becoming solely reliant on contemporary technology. Furthermore, Dr. Di Sessa emphasized that there are certain myths that must be dispelled about computational media to be able to educate from a perspective that facilitates genuine appreciation for new literacies based on them. Prevalent myths include, for example, the idea that technology is becoming better and better, therefore, there is not much to be learned about it; the use of advanced technology is sufficient for learning (one-way literacy); and the focus of teaching students how to program is so that they will get a job (vocationalism). Moreover, adopting a polemical stance, Dr. diSessa contends that we need to reconsider the popular trend in research that considers computational thinking (CT) as a literacy as CT is blind to real-knowledge (such as physics and mathematics).

Secondly, he explored the concept of re-mediation, emphasizing that representations change the intellectual world - what we can think and know (material intelligence). Dr. diSessa framed this idea within the historical context of how algebra transformed the field of science. He then seamlessly transitioned to the next two big ideas. Thirdly, he emphasized that computational representations change the nature of engagement and activity structure. Fourthly, he pointed out that the representations that computers afford through, for example programming, provide open toolsets of genetic resources for learners to adapt to specific topics. Finally, Dr. diSessa summarized his fifth big idea by explaining the LaDDER model - Layered, Distributed, Development of Education Resources. Using this acronym, he advised that educational toolsets and learning activities should be developed in a close but appropriate relationship with the classroom to maximally empower both teachers and students. He stressed the importance of granting teachers agency, that is, providing them with resources and ideas so that they can solve their own problems.

The conversation was furthered by a deeply thoughtful and challenging reaction from Dr. Brent Davis of University of Calgary. Dr. Davis interpreted Dr. diSessa's presentation as a manifesto, promoting computational literacy to do mathematics and science differently. However, he questioned the definition of computational literacy and, due to this dearth in understanding exactly what computational literacy is, he also questioned Dr. diSessa's activist perspective. Nevertheless, he challenged symposium participants to imagine their positions individually or collectively in this computational literacy discourse, whether as an observer describing or as an analyzer explaining or as an activist intervening. Davis also questioned the possibility of change in the near future, given the current state of institutions.

On the second day of the symposium, a discussion panel was held with a research focus, centering on the question: "How do computational thinking (CT) and mathematical thinking (MT) interact (in terms of knowledge, ways of thinking, and competencies)?" The panel, held virtually and livestreamed to attendees, was chaired by Dr. Ghislaine Gueudet (University Paris-Saclay, France). Panelists Dr. Paul Drijvers (Freudenthal Institute, Utrecht University, Netherlands), Dr. Eirini Geraniou (University College London, UK), and Dr. Elise Lockwood (Oregon State University, USA) brought their expertise together to relay knowledge, orchestrate an insightful discussion, and unearth the deep connections between CT and MT. As a precursor to the panel discussion, attendees answered three questions using an online app for responses, the Padlet, serving as an initial inventory of symposium attendees' experiences and views related to the interactions between CT and MT (focused on personal, societal, and educational perspectives). The results indicated that attendees had productive experiences of the interaction of CT and MT, for example, using coding software with students or even in their own working groups over the course of the symposium. However, the poll responses also revealed that there were still questions to be explored. One attendee pointed out the need for a common definition of computational thinking and mathematical thinking, and the distinction between the two, in order to answer the question. Against that backdrop each panelist put forward their augment.

Dr. Drijvers explored the relation between CT and MT, pointing out that we may begin to think about how they relate by inspecting where respective skills overlap (e.g. abstraction, generalization, decomposition, algorithms, problem solving etc.). Drawing on the works of Papert (1980) and Lodi (2020), he discerned computational thinking in mathematics education as structural problem-solving. This involves using thinking processes and skills common to CT and MT to solve or outsource mathematical problems to an external agent, whether that be another human or a machine. Giving several options of relational possibilities, Dr. Drijvers gave concrete examples of activities and tasks, both plugged and unplugged, showing the interaction of CT and MT.

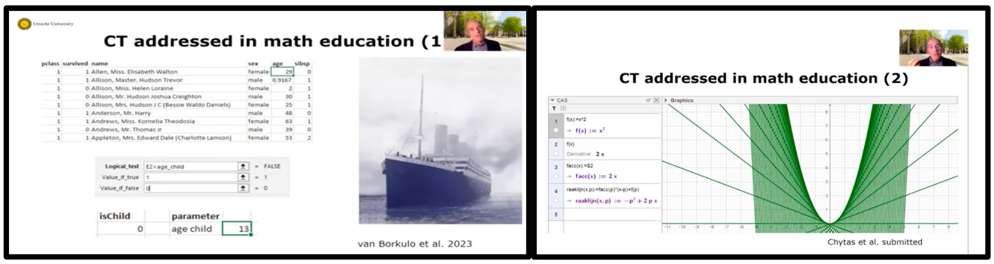

Figure 1. An illustration by Dr. Drijvers of plugged and unplugged activities.

In Dr. Drijvers' first example, students were provided with a spreadsheet containing Titanic passenger's data. Using CT and MT, students were tasked with employing specific variables to analyze the data by performing 'data moves.' The example shown involves counting the number of children aboard. In the second example, students were asked to input tangent lines to Geogebra in order to create a specific graph as the program's output. Dr. Drijvers concluded by highlighting that though CT and MT share common ground, further research is necessary to cultivate this shared territory.

Dr. Lockwood agreed that while CT and MT may have distinctive skills on their own, they also share a common ground. To illustrate her point, she highlighted her research in combinatorics (counting) problems, demonstrating that the existing limitations in helping students to connect counting processes to a set of outcomes is made easier by assisting them to develop simple programming tasks in Python. With the evidence that CT supports combinatory thinking, Dr. Lockwood was clear about her motivations to explore theoretically sound methods for researching how computational thinking can support what is known about mathematical thinking. Her closing statement prompted further exploration of how computational thinking and mathematical thinking support each other.

Lastly, Dr. Eirini Geraniou presented from her unique perspective as a researcher specializing in mathematics educators' perceptions of CT and how it interacts with mathematics education, as well as advocacy for CT integration in mathematics education. She began, like Dr. Drijvers, by pointing to varied but interrelated versions of the definition of CT over the years, following up with the question prompted by an assertion in Sands et al.'s (2018) research that CT is synonymous with mathematics that uses digital tools. Dr. Geraniou asks: "Are CT and MT only linked when using digital technologies?" Through her research, she has identified the ways in which mathematical and digital competencies are interconnected, developing the construct of MDC - Mathematical Digital Competency - pointing to key identifiers that demonstrate MDC in students (Geraniou & Jankvist, 2019).

Concluding remarks linked the panel discussion to the overall themes of the symposium, emphasizing the need for systemic changes in the field to advance equity and access. Attendees were also invited to participate by responding to additional poll questions using Padlet toward the end of the panel, discussing two thought-provoking vignettes - the 'Adventure Travel List Problem' and 'Math Ed in the Future.' These discussions speculatively explored how artificial intelligence might be applied in mathematics education. This culminating activity transitioned into a poster session during the lunch break.

Poster presenters represented the scope of the working groups, highlighting research and relevant practices aligned with the symposium's themes. In total, there were 19 posters: three focused on innovative practices, and 16 that were research-focused.

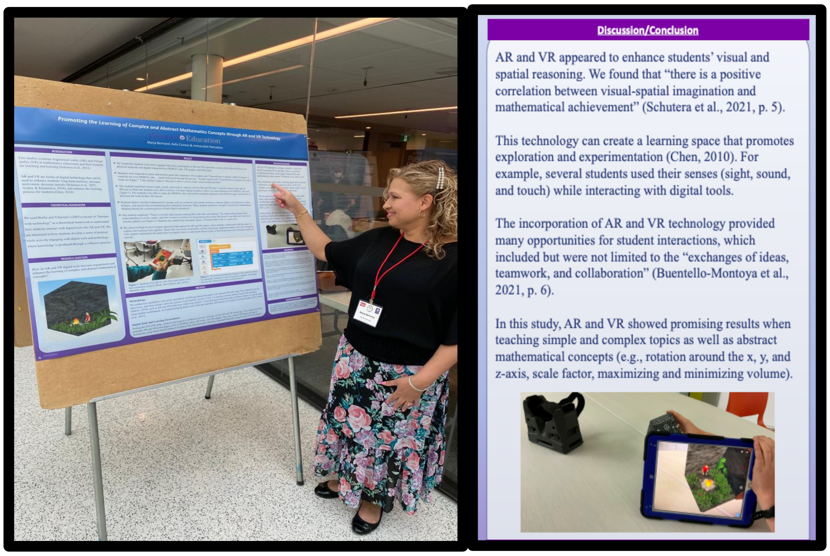

Figure 2. Marja Bertrand, a PhD Student at Western University, presents her poster Promoting the learning of complex and abstract mathematics concepts through AR and VR technology in the photo on the left. The Discussion/Conclusion section of her poster is displayed on the right.

On the evening of Day 2, the second keynote speaker, Dr. Gideon Christian from the Faculty of Law at the University of Calgary, delivered a presentation titled "Coded Bias: Decoding Racism in AI Technologies". Departing from the abstraction of coding and computational modeling and its many applications in everyday life, Dr. Christian focused on the potential unintended consequences of these technologies on regular people in society. Dr. Christian's years of experience in law using critical race theory (CRT) to identify barriers and racism in new and emerging technologies, especially in artificial intelligence (AI), came across quite vividly as he explained how AI can perpetuate racism. He preambled his presentation with an overview of CRT, dispelling common misconceptions about it before explaining how AI, one of the most transformative human inventions, is now subtly undermining the great progress that has been made in the area of civil rights to uproot elements such as of anti-black racism in society. Highlighting that the intent of CRT is to uncover racism embedded in structural institutional systems including law and policies, Dr. Christian keyed in on two aspects of AI that are biased toward people of color, particularly black people: (1) the use of AI in recidivism risk assessment, and (2) the use of AI in facial recognition systems, especially their use in law enforcement and in criminal justice systems. He noted that while the use of AI technologies in these areas has been touted as major innovation, their use comes with major concerns about their potential to perpetuate barriers and discrimination against black people and people of color. Dr. Christian provided several examples illustrating how black people are disproportionately represented in issues of misidentity in law enforcement and other areas in comparison to other groups. He explained that AI perpetuates barriers and racism when biased data is used to generate algorithms to train AI systems, emphasizing the concept of "garbage in, garbage out" in relation to data. In conclusion, Dr. Christian suggested that, given the high inaccuracy pertaining to some racial groups, it might be necessary to pause the use of AI tools on groups that are disproportionately affected. His presentation illuminated the need for educators and stakeholders who are connected to the development of AI systems to consider their role in maintaining structural biases, discrimination and racism.

The reaction to Dr. Chistian's keynote was done by Dr. Ron Eglash, professor in the School of Information at the University of Michigan with a secondary appointment in the Penny W. Stamps School of Art and Design. Drawing on his extensive experiences in culturally situated technological education and automation for generative justice, Dr. Eglash's reaction was an enriching extension of the discussion. While he endorsed Dr. Chistian's presentation, he emphasized the need to supplement an anti-bias framework with transformative justice and vision in computer science and mathematics education. Dr Eglash argued that if stakeholders solely focus on anti-bias in their educational work, this may lead to misunderstanding by students. He identified three challenges with exclusively projecting the negatives of computing: the belief that removing bias makes systems inherently fair; the perception that entering computing implies abandoning social justice issues; and the failure to challenge students to imaginatively develop new technologies for the purpose of transformative justice. Dr. Eglash, therefore, proposed adding aspects of ethno-mathematics and ethno-computing, heritage algorithms, and AI to critical tools may, from a transformative perspective, include different marginalized groups, thereby promoting equitable computing. To illustrate his point, he presented numerous examples of computational models developed with various ethnic groups including African-American and Indigenous communities.

Figure 3. An illustration by Dr. Ron Eglash of ethno-computing and heritage algorithms.

On the third and final day of the symposium, organizers collaborated with the Fields Math Ed. Forum to host a hybrid practice-focused panel discussion. The central question guiding the discussion was: "What are the challenges and opportunities of integrating coding in mathematics classrooms?" The panel was chaired by Dr. Dame Celia Hoyles (UCL Institute of Education, University College London, UK), with perspectives on the topics presented by Dr. George Gadanidis (Western University, Canada), Dr. Oh Nam Kwon (Seoul National University, South Korea), Dr. Simon Modeste (University of Montpellier, France), and Dr. Elena Prieto-Rodriguez (University of Newcastle, Australia).

Prof. Dame Hoyles opened the panel discussion by expressing her conviction, based on many years of experience, that despite challenges, there are potentials to integrate ideas of CT/ computational literacy, and programming into pedagogical practice to make a significant difference in mathematics learning. She emphasized that some of these potentials had already been mentioned in the symposium, such as coding making mathematics learning more dynamic and involved for students. Prof. Dame Hoyles further highlighted that coding is ubiquitous - a phenomenon all over the world - such that children are doing it at home, in their clubs, etc., with strides being made since the pandemic. As a segway to introducing the panelists, she pivoted to a discussion about the ways coding is being integrated into school curricula. Giving a brief historical background about the introduction of coding in mathematics education, referencing Seymour Papert's work on the consequential disruption, Prof. Dame Hoyles invited the panelists to make their presentations.

The diversity of the research panel allowed for deep and enriching perspective-sharing across contexts, made possible again through a hybrid synchronous format. Firstly, Dr. Kwon outlined a brief historical overview of coding and AI curriculum in South Korea in elementary and secondary education, concluding with several examples of coding education implemented in Korean classrooms. Dr. Modeste then outlined how students engage with coding and programming using project-based approaches in schools in France. Attendees were presented with specific examples, including video recordings of students working through a problem in Scratch. Dr. Gadanidis began his presentation with an example of a Grade 3-4 scratch puzzle, wherein students worked with variables to create spirals (pictured). Importantly, he emphasized that in such activities, the role of the teacher is not to explain, but to facilitate students explaining amongst themselves as they engage in play and exploration.

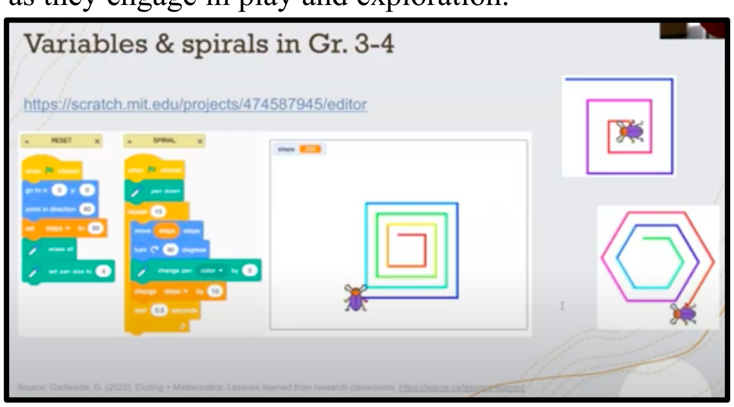

Figure 4. An illustration by Dr. George Gadanidis of a Grade 3-4 scratch puzzle.

Dr. Gadanidis proceeded to showcase more complex examples of programming using mathematics concepts as he transitioned to higher grades' curriculum and activities, using both Scratch and Python. After demonstrating potential practical uses, Dr. Gadanidis discussed how coding has become a new literacy as it is now mandatory in Ontario schools (serving over 1.5 million students). Drawing on Andrea diSessa's work, Dr. Gadanidis argued that coding may revitalize mathematics education by means of transforming how and what is taught in classrooms.

Dr. Prieto-Rodriguez provided an overview of her collaboration with Dr. Dame Hoyles and Dr. Noss on the integration of coding and computational thinking into mathematics curriculum in New South Wales, Australia using ScratchMaths. The project aimed to explore elementary teachers' perception of facilitating students' learning processes to develop mathematical ideas through coding, and how those perceptions change after eight weeks of professional learning. Dr. Prieto-Rodriguez highlighted that the results revealed statistically significant positive outcomes, impacting the knowledge and attitudes of both teachers and students. Her research was motivated by challenges she observed that hindered the national mandate of making CT and information systems mandatory for all Australian students from Kindergarten to Year 8.

After an opportunity for questions, Dr. Richard Noss, Emeritus Professor of Mathematics Education at University College London Institute of Education in the United Kingdom, provided a brief reaction. He emphasized that the symposium is a tangible representation emerging from years of contribution from many researchers who have theorized about the effectiveness of the computer for learning. Dr. Noss noted that the examples from Dr. Gadanidis' presentation suggested that the focus has shifted to *when* is the computer good for learning instead of *is* the computer good for learning. In general, he noted that it appears that significant progress has been made in research concerning the integration of CT in mathematics education. Building from there, Dr. Hoyles asked the following: Given that Noss's observation is true, how are we going to ensure that the progress in research changes the culture in school mathematics? This prompt segwayed the panel discussion into a lively Q & A involving attendees and panelists. The discourse drew out a diverse range of philosophical, pedagogical, and practical approaches and perspectives.

On the final day of the symposium, each working group had the opportunity to present their work to all attendees. As anticipated, the themes that emerged from the working group discussions were in alignment with the larger themes of the symposium. The picture attached shows a sample working group. Notably, Working Group A explored the notion that, for the elementary and early years, the focus of teaching should be building experiences (schema) within a sociocultural/societal framework. The aim of this is to provide students with future possibilities to think mathematically (proto-mathematics). According to the working group, coding enhances this experience; however, coding is not the mathematics itself.

In Working Group B, the focus was on sample coding and computational modeling activities. Guiding questions were used to prompt participants to reflect on the activities. One salient point from this group is that computational modeling is a way of bringing mathematicians, scientists, and computer scientists together - bringing mathematics into the mainstream of science - to address the complexities of today's most-pressing challenges.

Working Group C's activities were centered around developing a vision for Equity, Diversity and Inclusivity (EDI) using, as a lens, the 5 As (five affordances) of computational thinking (Agency, Access, Abstraction, Automation, and Audience) from Gadanidis' 2017 work. By the end of the three-day activities which included targeted small group engagement, they had developed an emerging framework for adapting coding activities to make them more inclusive. The framework includes three areas of considerations for reframing a task with respect to EDI principles: planning, implementing and assessing/building. At the planning stage, careful consideration is given to who the task is geared towards; at the implementation stage, the focus is on observing students to progressively create conditions for agency; at the assessing/building stage, the focus is on taking students from where they are to where they want to go.

Overall, the questions raised by the groups - pertaining to changes in tasks, tools for teaching, frameworks, classroom engagement, supporting pre-service and in-service teachers, and understanding mathematics in the context of CT - suggest that there is still much to be explored in research to elucidate the connection between CT and mathematics.

Finally, the summary of this report for the Professional Development (PD) day and CCMEME Symposium at Brock University reveals themes that suggest that the set objectives were met via participants' engagement in activities which includes coding workshops (with plugged-in and unplugged coding tasks), plenaries, panel discussions, a poster session, and working group discussions and reports.

Figure 6. Working group leaders and members at the symposium.