Using Robots to Develop Mathematical Objects-to-Think-With

Coding, Computational Modelling and Equity in Mathematics Education Symposium

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Objects-to-think-with

Using Papert's (1980) construct of "objects-to-think-with," we explore the assertion that the situated, movement-focused, and problem-driven spaces made available through digital technologies can enable the blending of competencies and concepts in ways that can amplify mathematical understanding. That is, we examine how objects-to-think-with might be leveraged to operate explicitly and implicitly as part of sense making in mathematics contexts.

Papert's notion of objects-to-think-with has "more to do with *structuring acts of moving* than with *acts of moving structures*" (Ng et al., 2018). That is, for Papert, objects-to-think-with served to "concretize the formal" (p. 21).

The objects-to-think-with that we focus on is number lines. For us, the interpretive power of number lines arises in the manner in which they always and already sit across enactive, iconic, and symbolic spaces of thinking.

Embodying concepts

We have offered many robotics workshops for 10- to 12-year-olds, focussing on developing spatial reasoning. One of the consistent findings in these workshops is that mathematics classes have not prepared students well to use decimal numbers flexibly. However, this changes when children learn decimals when programming robots to move.

In one 4th-grade class, we offered a robotics workshop at the start of the school year – before any formal instruction on decimals. The workshop began with the question, "How many wheel rotations are needed for the robot to travel 100 cm?" Each pair of students was provided with a meter stick, an EV3 Mindstorms robot, and an iPad loaded with EV3 Mindstorms software. They were also given initial directions on how to instruct the robot to move forward – namely, by entering the number of desired wheel rotations.

Emma and Jane began by guessing 100, which was wildly excessive.

For their second attempt, Emma and Jane entered 15, and they watched as the robot travelled more than double the length of the meter stick. For their third attempt, they entered "6," and the robot travelled just past the end of the meter stick.

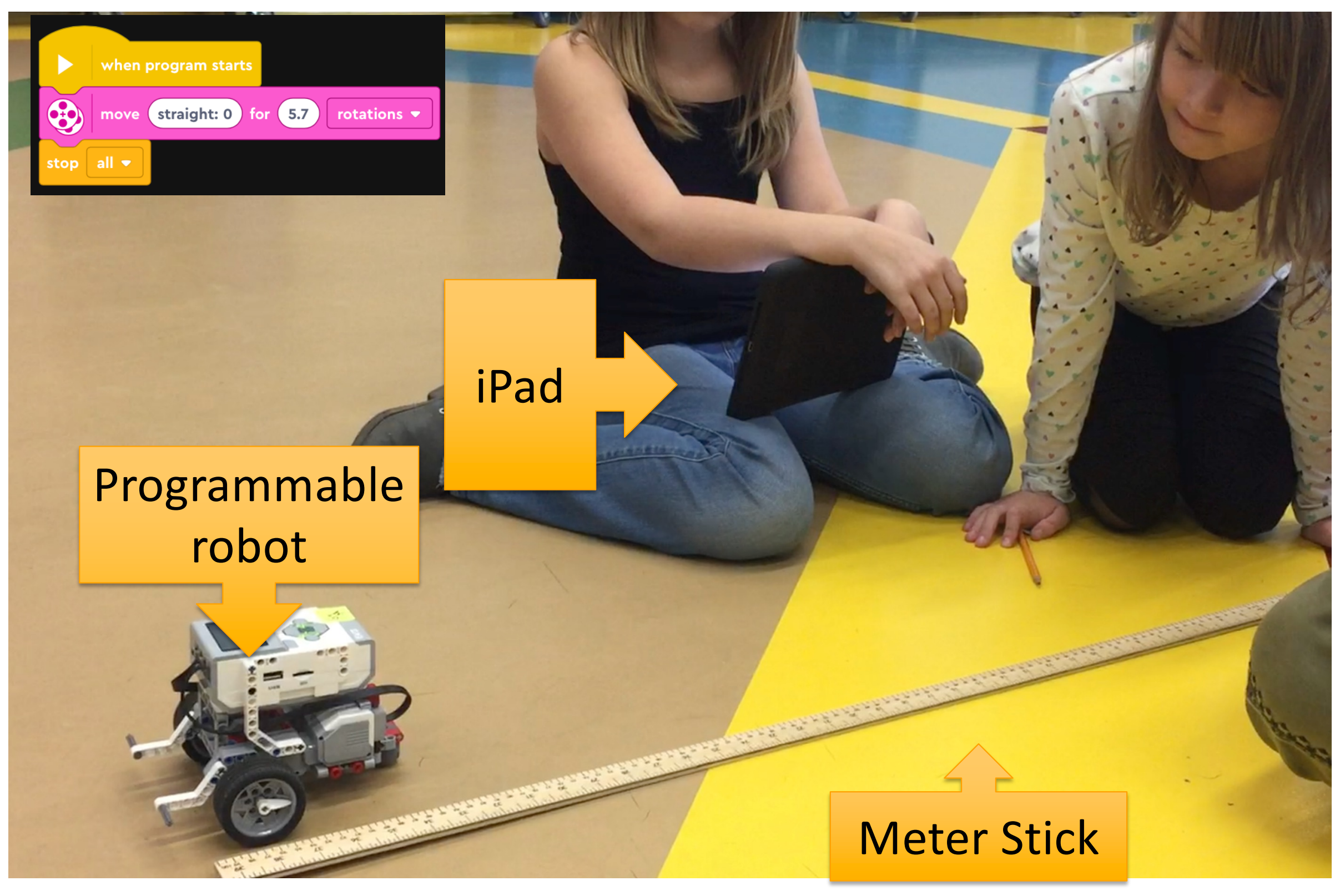
Since six turns were too many, the girls reasoned, "We need to go back one" to five rotations. However, when they entered "5," the robot did not quite reach the end of the meter stick. Aiming to help, the teacher asked, "What numbers are between 5 and 6?"

The girls looked a little confused, and one uttered, "Nothing," prompting the teacher to point to the locations on the meter stick to show that there were intervals between 5 and 6. She showed them how to enter decimals into the programming code. She explained, "Five decimal five is exactly halfway between five and six," prompting Emma and Jane to try "5.6." When the robot almost travelled to the end of the meter stick, they entered "5.7." To their delight, upon testing, the robot travelled 100 cm.

Sometime later, the concept of decimal fractions came up in mathematics class. The teacher enthusiastically reported that students were unexpectedly fluent with the concept, able to read measurements, combine amounts, and predict values with confidence and accuracy. In her experience of teaching Grade-4 mathematics, the concept had never been so "straightforward" for learners.

What does "number" mean for a 4th-grader who is competent with decimal fractions in mathematics class, but who cannot identify a number between two sequential wholes when tasked with coding a robot to move in a straight line?

Time and again in our research engagements, students have failed to transfer learnings about number (in particular, decimal fractions) from math classes to robotics workshops – a lapse, we believe, that signals an important difference between a "mathematical concept" and an "object-to-think-with."



Consider the number line, for instance, pasted on the classroom wall and peppered throughout classroom resources. The number line was certainly a familiar object to these students. With it, they had counted, compared, and calculated. But the familiarity developed through those activities did not appear to make the number line available as an object-to-think-with.

And, for the number line to be the ground of the activity – an object-to-think-with – it must be an embodied concept. It must give body to an idea that typically does not have one.

Discussion

We regard programmable robots and similar digital technologies as powerful means to support the learning of mathematics.

Embodying Concepts

Papert (1980) proposed the notion of "syntonic learning" to interpret his observations of children's use of Logo to program a turtle to trace out a circle. He noted their tendencies to perceive and characterize those movements in terms of their own actions and intentions. Contrasted with dissociated learning, syntonic learning is about engaging one's body and senses, aiming to develop a level of self-knowledge that enables the learner to transpose their own movement in space onto an object's movement or into a program for movement.

Phrased in terms of objects-to-think-with, such forms *extend* the knower – physically and/or cognitively. To illustrate physical extension, using Papert's example of gears, it is easy to imagine how playing with meshed gears might support insight into ratios, proportions, linear-causality, and other abstractions. As for cognitive extension, it is also easy to imagine how an abstracted understanding of simple gears might enable a knower to anticipate the behaviours of a compound gear.

Importantly, while we describe such cognitive acts as "extensions" in the previous paragraph, we suspect that there are actually multiple processes at play in such moments, including what others have labelled as "projection," "distribution," and "displacement." All these physical-and-cognitive actions are subsumed within the notion of "embodiment," which is perhaps a better choice than "extension" or "syntonic" for the purposes of this discussion.

To our understanding, body syntonic learning is consistent with such contemporary discourses on cognition as enactivism (Varela, Thompson, & Rosch, 1991), embodied cognition (Shapiro, 2011), extended cognition (Clark & Chalmers, 1998), inclusive materialism (de Freitas & Sinclair, 2014), and other perspectives that reject ego-centric models on cognition and lean more toward eco-centric interpretations in which cognitive activity is understood as embodied and embedded in and across multiple levels of organization. In general, these perspectives assume that human cognition simultaneously depends on having a biological body and on being part of a social, cultural, and/or ecological corpus. One's physical body defines possible movements and ranges of perception; the grander context in which one is embedded defines appropriate actions and the scopes of interpretive possibility. Across these frames, one's learning is understood to start with ranges of bodily motions and perceptions, but that learning is much elaborated through the technologies made available by the context.

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